Lesson 1-7: Perimeter, Circumference & Area

What is *PI*?

Seems like a pretty easy question, doesn't it? Everyone knows that *PI* is 3.1415926… And that is correct but not really what I asked. The number 3.1415926 is its value (actually it is an approximation of its value). What ***is*** *PI*? What does it ***mean***?

That's a little harder question isn't it? Let's hold on to that question for a bit; we'll come back to it.

Some more complex geometric figures

We've been talking about the basics of geometry: the point, line, and plane. We built some new things from them: the segment and ray. We discovered some properties of lines: parallel and skew. Let's do some more building…

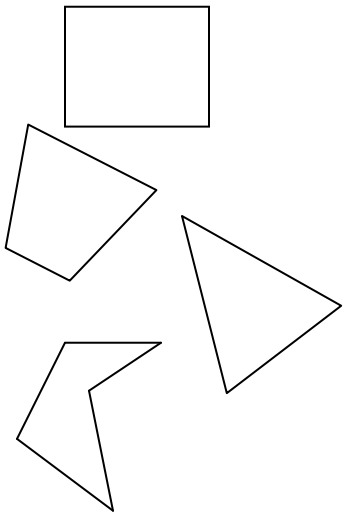
Polygons

A polygon is a closed geometric figure in a plane formed by connecting line segments endpoint to endpoint, each segment intersecting exactly two others.

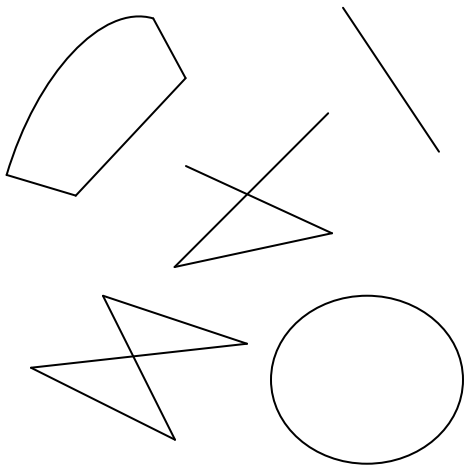
OK, that is a mouthful. Let's make it simple. A square is a polygon. It has no gaps (it is closed), its sides are segments that connect at the corners (endpoint to endpoint, exactly intersecting two others). A rectangle is another example.

Is a circle a polygon? No…it is made of a curve, not a line segment. Here are some more examples:

Yes



No



Circles

We all know what circles are. Let's give a circle a more formal (precise) definition:

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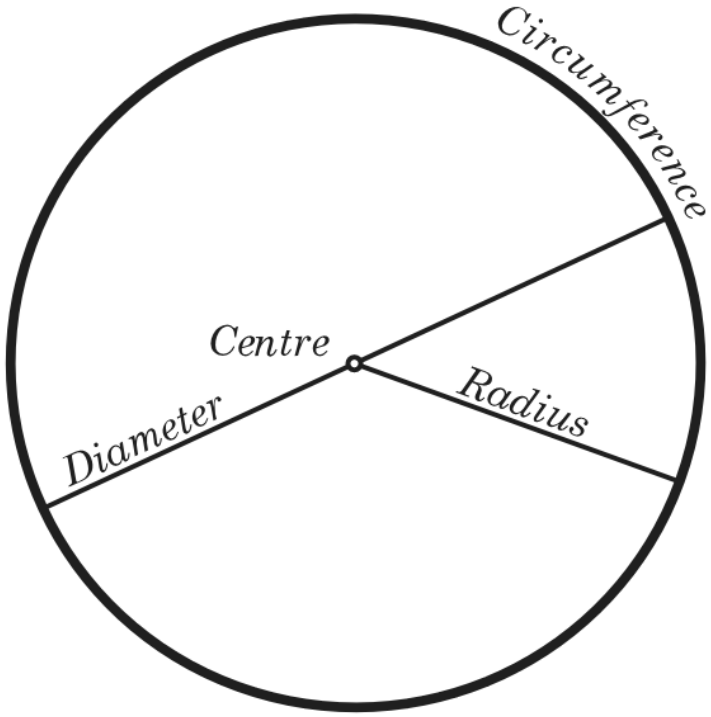
A circle is the set of all points in a plane that are a given distance (the *radius)* from a given point (the *center*).



Perimeter

Now that we have a working definition of our shapes, we can talk about things we can do with them. One thing that might be interesting is determining how big the shape is. The first measure of size for a shape is perimeter.

The perimeter of a polygon is the sum of the length of each side.

For now, we are going to restrict our focus to 2 basic polygons: squares and rectangles.

Perimeter *P* of a square with side length *s*: $P = 4s$

Perimeter *P* of a rectangle with base *b* and height *h*: $P = 2b + 2h$

Simple enough! It is basically the distance around the figure. What about a circle?

Circumference

Well, we don't use the term perimeter for the distance around a circle. The term we use is circumference. How do you measure that? I'm sure you can figure out a way but it certainly isn't as easy as a polygon which has straight sides. However, there is something about the circle that is far easier to measure. Its diameter or distance across. That you can easily do with a straight edge.

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Ok, so what … remember the first question I asked? What is *PI*? If you have the time, try this. Draw a decent sized circle. Measure the circumference (as accurately as you can), and measure the diameter. How do they relate? Play with it for a bit and see what you come up with. Draw a few more circles of different diameters and do the same. Do you see a relationship between the diameter and circumference that is common to all those circles?

If you play with it long enough you will discover a very interesting relationship. For every circle you can possibly draw, if you divide the circumference by the diameter, you will end up with the same number (give or take based on measurement error). That number is *PI*! Most simply put, the number *PI* is the ratio of any circle's circumference and diameter.

Circumference *C* of a circle of diameter *d* is: $C = \pi d = 2\pi r$

Area

Another way of determining the size of a shape is to determine its area. Area is basically how much surface space does the shape take up.

Area *A* of a square with side length *s*: $A = s^2$

Area *A* of a rectangle with base *b* and height *h*: $A = bh$

..and…

Area *A* of a circle of diameter *d* is: $A = \pi r^2$

Examples

Make sure you work through or at least look at the examples in the book. You should be able to do these easily.

Exploration

We've talked about congruent segments; we've talked about congruent angles. What do you suppose it would mean if I said two rectangles are congruent? Same size? Same shape? Same size and shape?

Postulate 1-9

Two figures are congruent if their areas are equal.

Exploration

Finding the area of a simple shape such as a square, rectangle or circle is not difficult. But suppose we have a complicated shape? How would we find its area?

Consider the following shape and try to figure out how you could determine its area:

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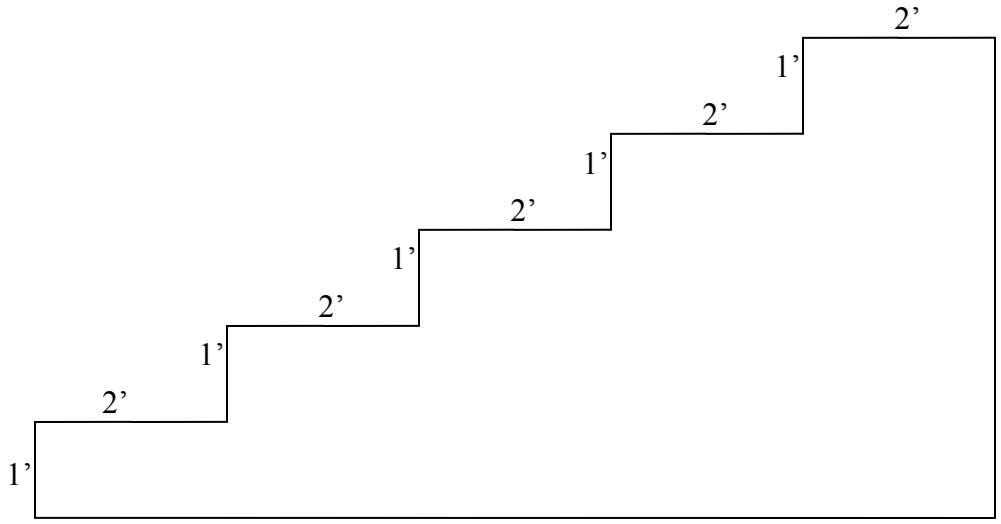
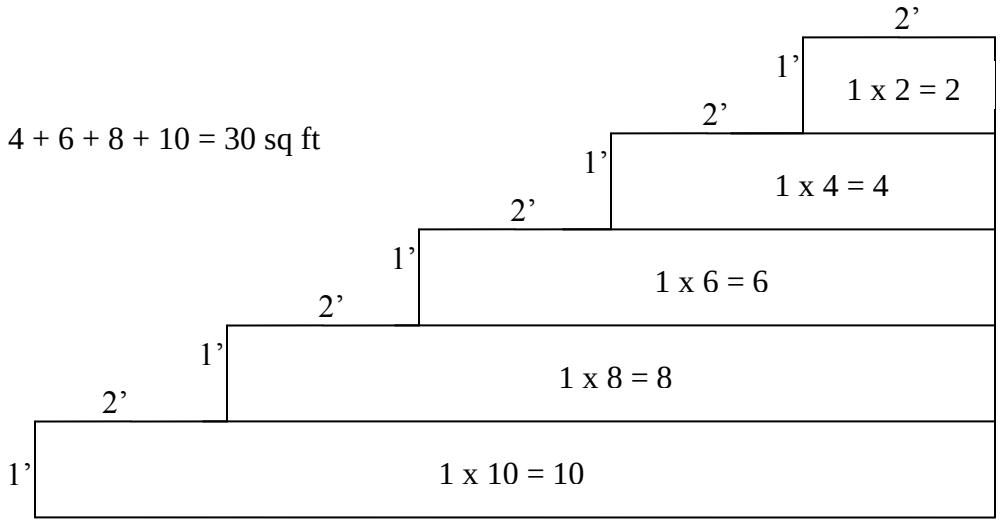


Do we have tools at our disposal that can help us solve this problem? We’re trying to determine area right? For what figures do we know how to calculate area?

What if we broke this complicated shape into smaller sub-shapes that we know how to deal with? Now we’d have a bunch of rectangles and we know how to find area for a rectangle!

2 + 4 + 6 + 8 + 10 = 30 sq ft



Postulate 1-10

The area of a region is the sum of the areas of **non-overlapping** parts.

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Formulas from today

Perimeter *P* of a square with side length *s*: $P = 4s$

Perimeter *P* of a rectangle with base *b* and height *h*: $P = 2b + 2h$

Area *A* of a square with side length *s*: $A = s^2$

Area *A* of a rectangle with base *b* and height *h*: $A = bh$

Area *A* of a circle of diameter *d* is: $A = \pi r^2$

Assign homework

p. 55 1-41 odd, 50, 54-57, 61, 68, 69, 72